Mechanical Engineering Examples of the Secant Method
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Example 1

A trunnion has to be cooled before it is shrink fitted into a steel hub.

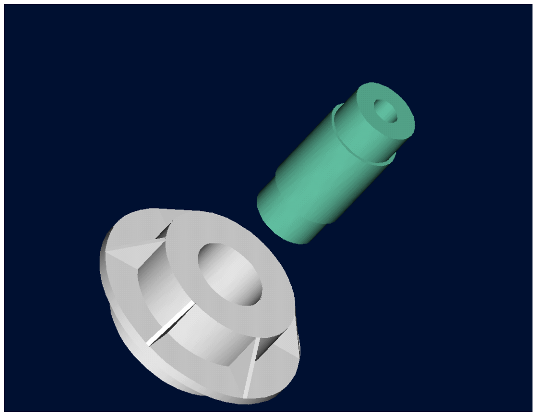

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the temperature T_f to which the trunnion has to be cooled to obtain the desired contraction is given by

$$f\left(T_f\right) = -0.50598\times10^{-10}T_f^3 + 0.38292\times10^{-7}T_f^2 + 0.74363\times10^{-4}T_f + 0.88318\times10^{-2} = 0$$

Use the secant method of finding roots of equations to find the temperature T_f to which the trunnion has to be cooled to. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and

the number of significant digits at least correct at the end of each iteration.

Solution

Let us take the initial guesses of the root of $f\left(T_f\right)=0$ as $T_{f,-1} = -110$ and $T_{f,0} = -130$.

<u>Iteration 1</u>
The estimate of the root is

$$T_{f,1} = T_{f,0} - \frac{f(T_{f,0})(T_{f,0} - T_{f,-1})}{f(T_{f,0}) - f(T_{f,-1})}$$

$$f(T_{f,0}) = -0.50598\times10^{-10}T_{f,0}^3 + 0.38292\times10^{-7}T_{f,0}^2 + 0.74363\times10^{-4}T_{f,0} + 0.88318\times10^{-2}$$

$$= -0.50598\times10^{-10}(-130)^3 + 0.38292\times10^{-7}(-130)^2 + 0.74363\times10^{-4}(-130) + 0.88318\times10^{-2}$$

$$= -7.7091\times10^{-5}$$

$$f(T_{f,-1}) = -0.50598\times10^{-10}T_{f,-1}^3 + 0.38292\times10^{-7}T_{f,-1}^2 + 0.74363\times10^{-4}T_{f,-1} + 0.88318\times10^{-2}$$

$$= -0.50598\times10^{-10}(-110)^3 + 0.38292\times10^{-7}(-110)^2 + 0.74363\times10^{-4}(-110) + 0.88318\times10^{-2}$$

$$= 1.1825\times10^{-3}$$

$$T_{f,1} = -130 - \frac{(-7.7091\times10^{-5})(-130-(-110))}{(-7.7091\times10^{-5}) - (1.1825\times10^{-3})}$$

$$= -128.78$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 1 is

$$|\in_a| = \left|\frac{T_{f,1} - T_{f,0}}{T_{f,1}}\right| \times 100$$

$$= \left|\frac{-128.78 - (-130)}{-128.78}\right| \times 100$$

$$= 0.95051\%$$

The number of significant digits at least correct is 1, because the absolute relative approximate error is less than 5%.

Iteration 2

The estimate of the root is

$$T_{f,2} = T_{f,1} - \frac{f(T_{f,1})(T_{f,1} - T_{f,0})}{f(T_{f,1}) - f(T_{f,0})}$$

$$f(T_{f,1}) = -0.50598 \times 10^{-10} T_{f,1}^3 + 0.38292 \times 10^{-7} T_{f,1}^2 + 0.74363 \times 10^{-4} T_{f,1} + 0.88318 \times 10^{-2}$$

$$= -0.50598 \times 10^{-10}(-128.78)^3 + 0.38292 \times 10^{-7}(-128.78)^2$$
$$+ 0.74363 \times 10^{-4}(-128.78) + 0.88318 \times 10^{-2}$$

$$= -1.3089 \times 10^{-6}$$

$$T_{f,2} = -128.78 - \frac{(-1.3089 \times 10^{-6})(-128.78 - (-130))}{(-1.3089 \times 10^{-6}) - (-7.7091 \times 10^{-5})}$$

$$= -128.75$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 2 is

$$|\in_a| = \left| \frac{T_{f,2} - T_{f,1}}{T_{f,2}} \right| \times 100$$

$$= \left| \frac{-128.75 - (-128.78)}{-128.75} \right| \times 100$$

$$= 0.016419\%$$

The number of significant digits at least correct is 3.

<u>Iteration 3</u>
The estimate of the root is

$$T_{f,3} = T_{f,2} - \frac{f(T_{f,2})(T_{f,2} - T_{f,1})}{f(T_{f,2}) - f(T_{f,1})}$$

$$f(T_{f,2}) = -0.50598 \times 10^{-10} T_{f,2}^3 + 0.38292 \times 10^{-7} T_{f,2}^2 + 0.74363 \times 10^{-4} T_{f,2} + 0.88318 \times 10^{-2}$$

$$= -0.50598 \times 10^{-10}(-128.75)^3 + 0.38292 \times 10^{-7}(-128.75)^2$$
$$+ 0.74363 \times 10^{-4}(-128.75) + 0.88318 \times 10^{-2}$$

$$= 1.5241 \times 10^{-9}$$

$$T_{f,3} = -128.75 - \frac{\left(1.5241\times10^{-5}\right)\left(-128.75-(-128.78)\right)}{\left(1.5241\times10^{-9}\right)-\left(-1.3089\times10^{-6}\right)}$$

$$= -128.75$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 3 is

$$|\epsilon_a| = \left|\frac{T_{f,3}-T_{f,2}}{T_{f,3}}\right|\times100$$

$$= \left|\frac{-128.75-(-128.75)}{-128.75}\right|\times100$$

$$= 1.9097\times10^{-5}\%$$

The number of significant digits at least correct is 6, because the absolute relative approximate error is less than 0.00005%.