Mechanical Engineering Example of Gaussian Elimination

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Example 1

A trunnion of diameter $12.363"$ has to be cooled from a room temperature of $80\,°\mathrm{F}$ before it is shrink fitted into a steel hub (Figure 1).

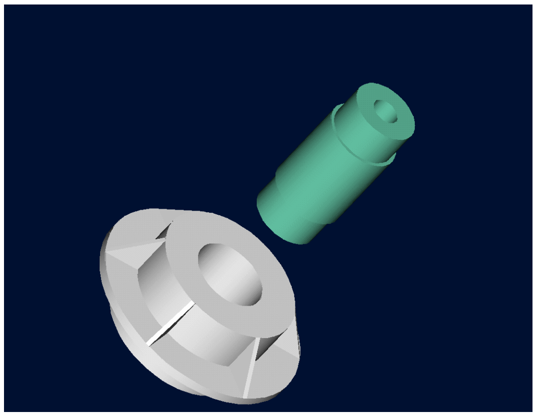

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction ΔD of the trunnion in a dry-ice/alcohol mixture (boiling temperature is $-108\,°\mathrm{F}$) is given by

$$\Delta D = 12.363 \int_{80}^{-108} \alpha(T)\,dT$$

The equation for the thermal expansion coefficient, $\alpha = a_1 + a_2 T + a_3 T^2$, is obtained using regression analysis where the constants of the model are found by solving the following simultaneous linear equations.

$$\begin{bmatrix} 24 & -2860 & 7.26\times10^{5} \\ -2860 & 7.26\times10^{5} & -1.86472\times10^{8} \\ 7.26\times10^{5} & -1.86472\times10^{8} & 5.24357\times10^{10} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1.057\times10^{-4} \\ -1.04162\times10^{-2} \\ 2.56799 \end{bmatrix}$$

Find the values of a_1, a_2, and a_3 using naïve Gauss elimination.

Source URL: http://numericalmethods.eng.usf.edu/
Saylor URL: http://www.saylor.org/courses/me205/

Solution

Forward Elimination of Unknowns

Since there are three equations, there will be two steps of forward elimination of unknowns.

First step

Divide Row 1 by 24 and then multiply it by −2860, that is, multiply Row 1 by $-2860/24 = -119.17$.

$$\text{Row 1} \times (-119.17) = \begin{bmatrix} -2860 & 3.4082 \times 10^{5} & -8.6515 \times 10^{7} \end{bmatrix} \qquad \begin{bmatrix} -0.012596 \end{bmatrix}$$

Subtract the result from Row 2 to get

$$\begin{bmatrix} 24 & -2860 & 7.26 \times 10^{5} \\ 0 & 3.8518 \times 10^{5} & -9.9957 \times 10^{7} \\ 7.26 \times 10^{5} & -1.86472 \times 10^{8} & 5.243 \times 10^{10} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1.057 \times 10^{-4} \\ 2.1797 \times 10^{-3} \\ 2.56799 \end{bmatrix}$$

Divide Row 1 by 24 and then multiply it by 7.26×10^{5}, that is, multiply Row 1 by $7.26 \times 10^{5}/24 = 30250$.

$$\text{Row 1} \times (30250) = \begin{bmatrix} 7.26 \times 10^{5} & 8.6515 \times 10^{7} & 2.1962 \times 10^{10} \end{bmatrix} \qquad \begin{bmatrix} 3.1974 \end{bmatrix}$$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 24 & -2860 & 7.26 \times 10^{5} \\ 0 & 3.8518 \times 10^{5} & -9.9957 \times 10^{7} \\ 0 & -9.9957 \times 10^{7} & 3.04742 \times 10^{10} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1.057 \times 10^{-4} \\ 2.1797 \times 10^{-3} \\ -0.62944 \end{bmatrix}$$

Second step

We now divide Row 2 by 3.8518×10^{5} and then multiply it by -9.9957×10^{7}, that is, multiply Row 2 by $-9.9957 \times 10^{7}/3.8518 \times 10^{5} = -259.50$.

$$\text{Row } 2 \times (-259.50) = \begin{bmatrix} 0 & -9.9957\times 10^{7} & 2.5939\times 10^{10} \end{bmatrix} \quad \begin{bmatrix} -0.56565 \end{bmatrix}$$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 24 & -2860 & 7.26\times 10^{5} \\ 0 & 3.8518\times 10^{5} & -9.9957\times 10^{7} \\ 0 & 0 & 4.5349\times 10^{9} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1.057\times 10^{-4} \\ 2.1797\times 10^{-3} \\ -6.3788\times 10^{-2} \end{bmatrix}$$

Back Substitution

From the third equation,

$$4.5349\times 10^{9} a_3 = -6.3788\times 10^{-2}$$

$$a_3 = \frac{-6.3788\times 10^{-2}}{4.5349\times 10^{9}}$$

$$= -1.4066\times 10^{-11}$$

Substituting the value of a_3 in the second equation,

$$3.8518\times 10^{5} a_2 + \left(-9.9957\times 10^{7}\right)a_3 = 2.1797\times 10^{-3}$$

$$a_2 = \frac{2.1797\times 10^{-3} - \left(-9.9957\times 10^{7}\right)a_3}{3.8518\times 10^{5}}$$

$$= \frac{2.1797\times 10^{-3} - \left(-9.9957\times 10^{7}\right)\times\left(-1.4066\times 10^{-11}\right)}{3.8518\times 10^{5}}$$

$$= 2.0087\times 10^{-9}$$

Substituting the values of a_2 and a_3 in the first equation,

$$24a_1 + (-2860)a_2 + 7.26\times 10^{5} a_3 = 1.057\times 10^{-4}$$

$$a_1 = \frac{1.057\times 10^{-4} - (-2860)a_2 - 7.26\times 10^{5} a_3}{24}$$

$$= \frac{1.057\times10^{-4} - (-2860)\times(2.0087\times10^{-9}) - 7.26\times10^{5}\times(-1.4066\times10^{-11})}{24}$$

$$= 5.0690\times10^{-6}$$

Hence the solution vector is

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 5.0690\times10^{-6} \\ 2.0087\times10^{-9} \\ -1.4066\times10^{-11} \end{bmatrix}$$