Industrial Engineering Example of Linear Regression
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Example 1

As machines are used over long periods of time, the output product can get off target. Below is the average value of how much off target a product is getting manufactured as a function of machine use.

Table 1 Off target value as a function of machine use.

Hours of Machine Use, t	30	33	34	35	39	44	45
Millimeters Off Target, h	1.10	1.21	1.25	1.23	1.30	1.40	1.42

Regress the data to $h = a_0 + a_1 t$. Find when the product will be 2 mm off target.

Solution

Table 2 shows the summations needed for the calculation of the constants of the regression model.

Table 2 Tabulation of data for calculation of needed summations.

I	t	h	t^2	$t \times h$
−	Hours	Millimeters	Hours^2	Millimeter-Hour
1	30	1.10	900	33
2	33	1.21	1089	39.93
3	34	1.25	1156	42.50
4	35	1.23	1225	43.05
5	39	1.30	1521	50.70
6	44	1.40	1936	61.6

7	45	1.42	2025	63.9
$\sum_{i=1}^{7}$	260	8.91	9852	334.68

$$n = 7$$

$$a_1 = \frac{n\sum_{i=1}^{7} t_i h_i - \sum_{i=1}^{7} t_i \sum_{i=1}^{7} h_i}{n\sum_{i=1}^{7} t_i^2 - \left(\sum_{i=1}^{7} t_i\right)^2}$$

$$= \frac{7(334.68) - (260)(8.91)}{7(9852) - (260)^2}$$

$$= 0.019179 \text{ mm-h}$$

$$\bar{h} = \frac{\sum_{i=1}^{7} h_i}{n}$$

$$= \frac{8.91}{7}$$

$$= 1.2729 \text{ mm}$$

$$\bar{t} = \frac{\sum_{i=1}^{7} t_i}{n}$$

$$= \frac{260}{7}$$

$$= 37.143 \text{ h}$$

$$a_0 = \bar{h} - a_1 \bar{t}$$

$$= 1.2729 - (0.019179)(37.143)$$

$$= 0.56050 \text{ mm-h}$$

$$h = 0.56050 + 0.019179t$$

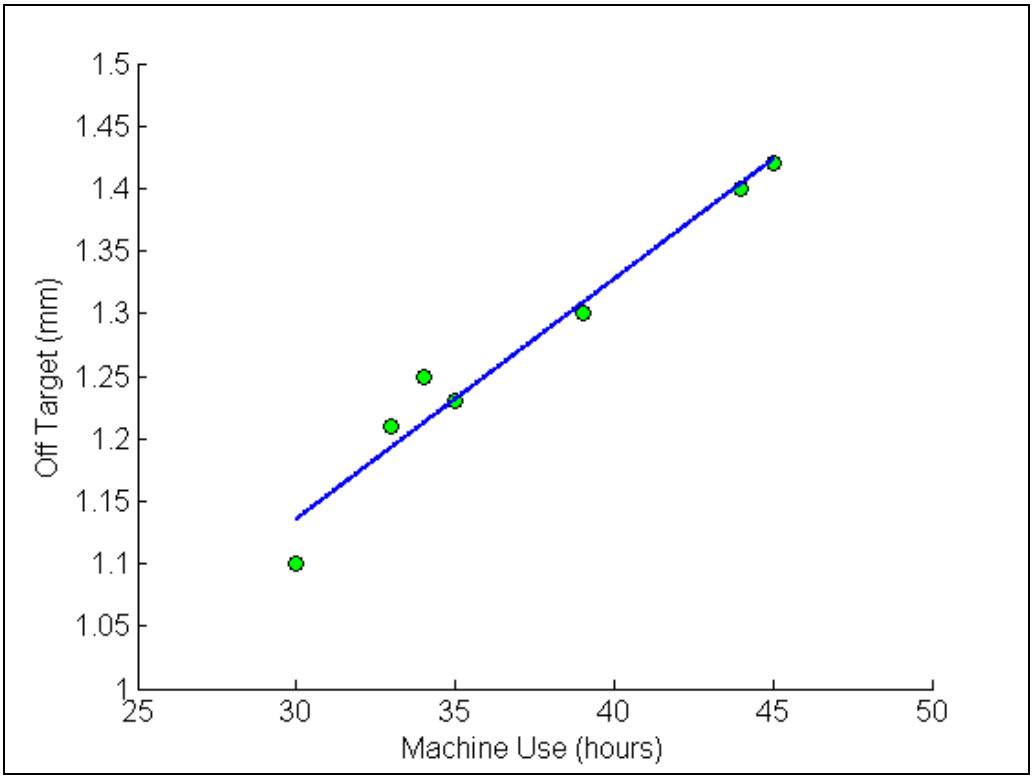

Figure 1 Linear regression of hours of use vs. millimeters off target.

Solving for $h = 2$ mm, the regression model is $h = 0.56050 + 0.019179t$

$$2 = 056050 + 0.019179t$$

$$t = \frac{2 - 0.56050}{0.019179}$$

$t = 75.056$ hours