

Multiple-Choice Test – Non-Linear Regression
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1. When using the linearized data model to find the constants of the regression model $y = ae^{bx}$ to best fit $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, the sum of the square of the residuals that is minimized is

(A) $\sum_{i=1}^n (y_i - ae^{bx_i})^2$

(B) $\sum_{i=1}^n (\ln(y_i) - \ln(a) - bx_i)^2$

(C) $\sum_{i=1}^n (y_i - \ln(a) - bx_i)^2$

(D) $\sum_{i=1}^n (\ln(y_i) - \ln(a) - b \ln(x_i))^2$

2. It is suspected from theoretical considerations that the rate of water flow from a firehouse is proportional to some power of the nozzle pressure. Assume pressure data is more accurate. You are transforming the data.

Flow rate, F (gallons/min)	96	129	135	145	168	235
Pressure, p (psi)	11	17	20	25	40	55

The exponent of the nozzle pressure in the regression model $F = ap^b$ most nearly is

- (A) 0.49721
- (B) 0.55625
- (C) 0.57821
- (D) 0.67876
3. The linearized data model for the stress-strain curve $\sigma = k_1 \varepsilon e^{-k_2 \varepsilon}$ for concrete in compression, where σ is the stress and ε is the strain, is
- (A) $\ln(\sigma) = \ln(k_1) + \ln(\varepsilon) - k_2 \varepsilon$
- (B) $\ln\left(\frac{\sigma}{\varepsilon}\right) = \ln(k_1) - k_2 \varepsilon$



$$(C) \ln\left(\frac{\sigma}{\varepsilon}\right) = \ln(k_1) + k_2\varepsilon$$

$$(D) \ln(\sigma) = \ln(k_1\varepsilon) - k_2\varepsilon$$

4. In nonlinear regression, finding the constants of the model requires solving simultaneous nonlinear equations. However in the exponential model $y = ae^{bx}$ that is best fit to $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, the value of b can be found as a solution of a nonlinear equation. That equation is given by

$$(A) \sum_{i=1}^n y_i x_i e^{bx_i} - \sum_{i=1}^n y_i e^{bx_i} \sum_{i=1}^n x_i = 0$$

$$(B) \sum_{i=1}^n y_i x_i e^{bx_i} - \frac{\sum_{i=1}^n y_i e^{bx_i}}{\sum_{i=1}^n e^{2bx_i}} \sum_{i=1}^n x_i e^{2bx_i} = 0$$

$$(C) \sum_{i=1}^n y_i x_i e^{bx_i} - \frac{\sum_{i=1}^n y_i e^{bx_i}}{\sum_{i=1}^n e^{2bx_i}} \sum_{i=1}^n e^{bx_i} = 0$$

$$(D) \sum_{i=1}^n y_i e^{bx_i} - \frac{\sum_{i=1}^n y_i e^{bx_i}}{\sum_{i=1}^n e^{2bx_i}} \sum_{i=1}^n x_i e^{2bx_i} = 0$$

5. There is a functional relationship between the mass density ρ of air and the altitude h above the sea level.

Altitude above sea level, h (km)	0.32	0.64	1.28	1.60
Mass Density, ρ (kg/m^3)	1.15	1.10	1.05	0.95

In the regression model $\rho = k_1 e^{-k_2 h}$, the constant k_2 is found as $k_2 = 0.1315$.

Assuming the mass density of air at the top of the atmosphere is $1/1000^{\text{th}}$ of the mass density of air at sea level. The altitude in kilometers of the top of the atmosphere most nearly is

- (A) 46.2
(B) 46.6



- (C) 49.7
- (D) 52.5

6. A steel cylinder at 80° F of length 12" is placed in a commercially available liquid nitrogen bath (−315° F). If the thermal expansion coefficient of steel behaves as a second order polynomial that is a function of temperature and the polynomial is found by regressing the data below,

Temperature, T (°F)	Thermal expansion Coefficient, α (μ in/in/°F)
−320	2.76
−240	3.83
−160	4.72
−80	5.43
0	6.00
80	6.47

the reduction in the length of the cylinder in inches most nearly is

- (A) 0.0219
- (B) 0.0231
- (C) 0.0235
- (D) 0.0307

