Electrical Engineering Examples of Nonlinear Models for Regression
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Example 1

To be able to draw road networks from aerial images, light intensities are measured at different pixel locations. The following intensities are given as a function of pixel location.

Table 1 Light intensities as a function of pixel location.

Pixel Location, k	Intensity, y
−3	119
−2	165
−1	231
0	243
1	244
2	214
3	136

Regress the above data to a second order polynomial

$$y = a_0 + a_1 k + a_2 k^2$$

Solution

Table 2 shows the summations needed for the calculation of the constants of the regression model.

Table 2 Summations for calculating constants of model.

i	Pixel Location, k	Intensity, y	k^2	k^3	k^4	$k \times y$	$k^2 \times y$
1	−3	119	9	−27	81	−357	1071
2	−2	165	4	−8	16	−330	660
3	−1	231	1	−1	1	−231	231
4	0	243	0	0	0	0	0
5	1	244	1	1	1	244	244
6	2	214	4	8	16	428	856
7	3	136	9	27	81	408	1224
$\sum_{i=1}^{7}$	0	1352	28	0	196	162	4286

$y = a_0 + a_1 k + a_2 k^2$ is the quadratic relationship between the pixel location and intensity where the coefficients a_0, a_1, a_2 are found as follows

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} k_i\right) & \left(\sum_{i=1}^{n} k_i^2\right) \\ \left(\sum_{i=1}^{n} k_i\right) & \left(\sum_{i=1}^{n} k_i^2\right) & \left(\sum_{i=1}^{n} k_i^3\right) \\ \left(\sum_{i=1}^{n} k_i^2\right) & \left(\sum_{i=1}^{n} k_i^3\right) & \left(\sum_{i=1}^{n} k_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} k_i y_i \\ \sum_{i=1}^{n} k_i^2 y_i \end{bmatrix}$$

$n = 7$

$$\sum_{i=1}^{7} k_i = 0$$

$$\sum_{i=1}^{7} k_i^2 = 28$$

$$\sum_{i=1}^{7} k_i^3 = 0$$

$$\sum_{i=1}^{7} k_i^4 = 196$$

$$\sum_{i=1}^{7} y_i = 1352$$

$$\sum_{i=1}^{7} k_i y_i = 162$$

$$\sum_{i=1}^{7} k_i^2 y_i = 4286$$

We have

$$\begin{bmatrix} 7 & 0 & 28 \\ 0 & 28 & 0 \\ 28 & 0 & 196 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1352 \\ 162 \\ 4286 \end{bmatrix}$$

Solve the above system of simultaneous linear equations, we get

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 246.57 \\ 5.7857 \\ -13.357 \end{bmatrix}$$

The polynomial regression model is

$$P = a_0 + a_1 m + a_2 m^2$$
$$= 246.57 + 5.7857m - 13.357m^2$$

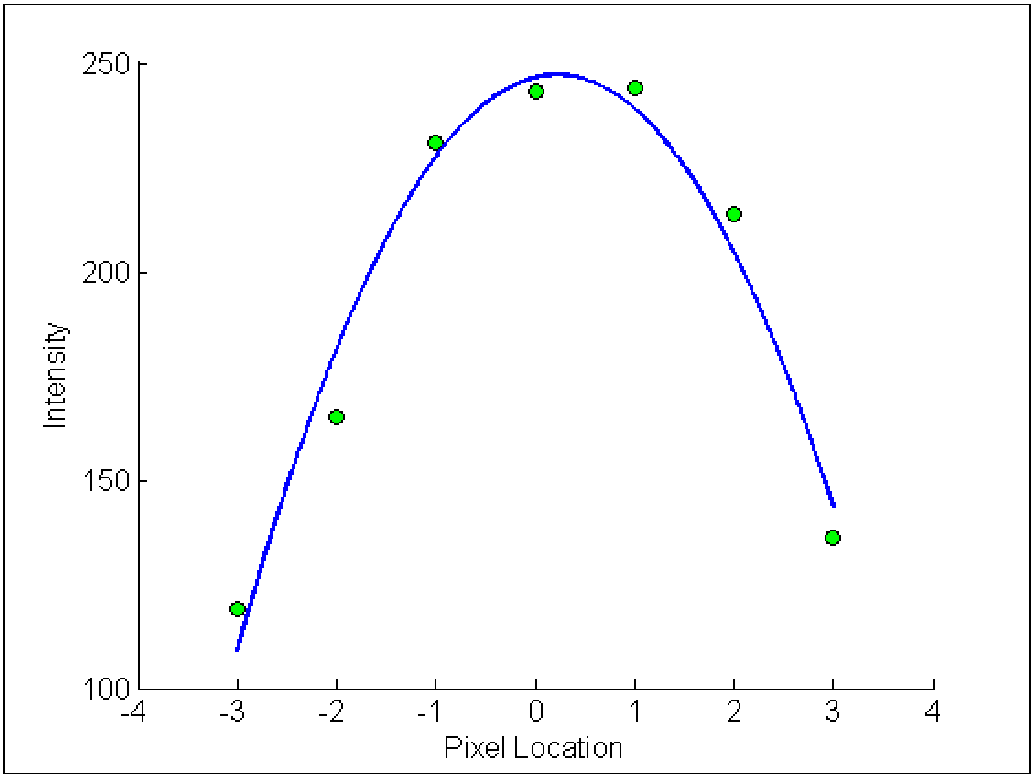

Figure 1 Second order polynomial regression model for intensity as a function of pixel location.