Industrial Engineering Examples for Euler's Method for Ordinary Differential Equations
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Example 1

The open loop response, that is, the speed of the motor to a voltage input of 20 V, assuming a system without damping is

$$20 = (0.02)\frac{dw}{dt} + (0.06)w.$$

If the initial speed is zero $(w(0) = 0)$, and using Euler's method, what is the speed at $t = 0.8\,\text{s}$? Assume a step size of $h = 0.4$ s.

Solution

$$\frac{dw}{dt} = 1000 - 3w$$

$$f(t, w) = 1000 - 3w$$

The Euler's method reduces to

$$w_{i+1} = w_i + f(t_i, w_i)h$$

For $i = 0$, $t_0 = 0$, $w_0 = 0$

$$w_1 = w_0 + f(t_0, w_0)h$$

$$= 0 + f(0,0)\times 0.4$$

$$= 0 + (1000 - 3\times(0))\times 0.4$$

$$= 0 + 1000\times 0.4$$

$$= 400\ \text{rad/s}$$

w_1 is the approximate speed of the motor at

$$t = t_1 = t_0 + h = 0 + 0.4 = 0.4\,\text{s}$$

$$w(0.4) \approx w_1 = 400 \text{ rad/s}$$

For $i = 1$, $t_1 = 0.4\,\text{s}$, $w_1 = 400$

$$w_2 = w_1 + f(t_1, w_1)h$$

$$= 400.00 + f(0.4, 400) \times 0.4$$

$$= 400.00 + (1000 - 3 \times 400) \times 0.4$$

$$= 400 + (-200) \times 0.4$$

$$= 320 \text{ rad/s}$$

w_2 is the approximate speed of the motor at

$$t = t_2 = t_1 + h = 0.4 + 0.4 = 0.8\,\text{s}$$

$$w(0.8) \approx w_2 = 320 \text{ rad/s}$$

Figure 1 compares the exact solution with the numerical solution from Euler's method for the step size of $h = 0.4\,\text{s}$.

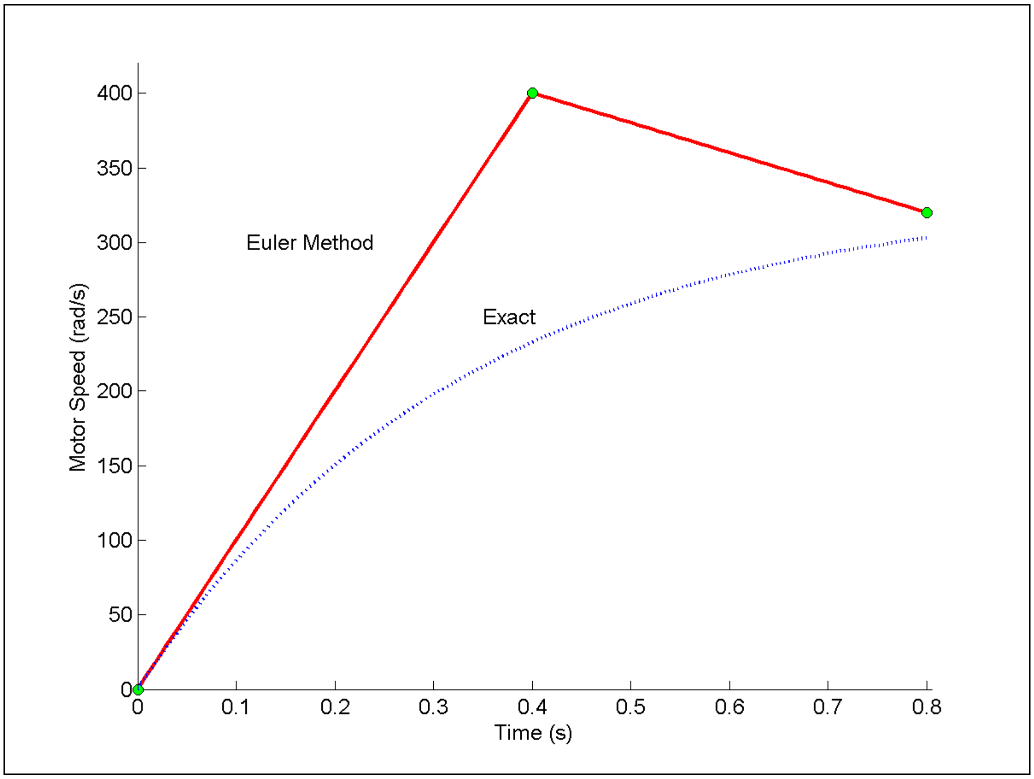

Figure 1 Comparing exact and Euler's method.

The problem was solved again using smaller step sizes. The results are given below in Table 1.

Table 1 Speed of motor at 0.8 seconds as a function of step size, h.

Step size, h	$w(0.8)$	E_t	$\lvert\in_t\rvert\,\%$
0.8	800	−496.91	163.95
0.4	320	−16.906	5.5778
0.2	324.8	−21.706	7.1615

0.1	314.18	−11.023	3.6370
0.05	308.58	−5.4890	1.8110

Figure 2 shows how the speed of the motor varies as a function of time for different step sizes.

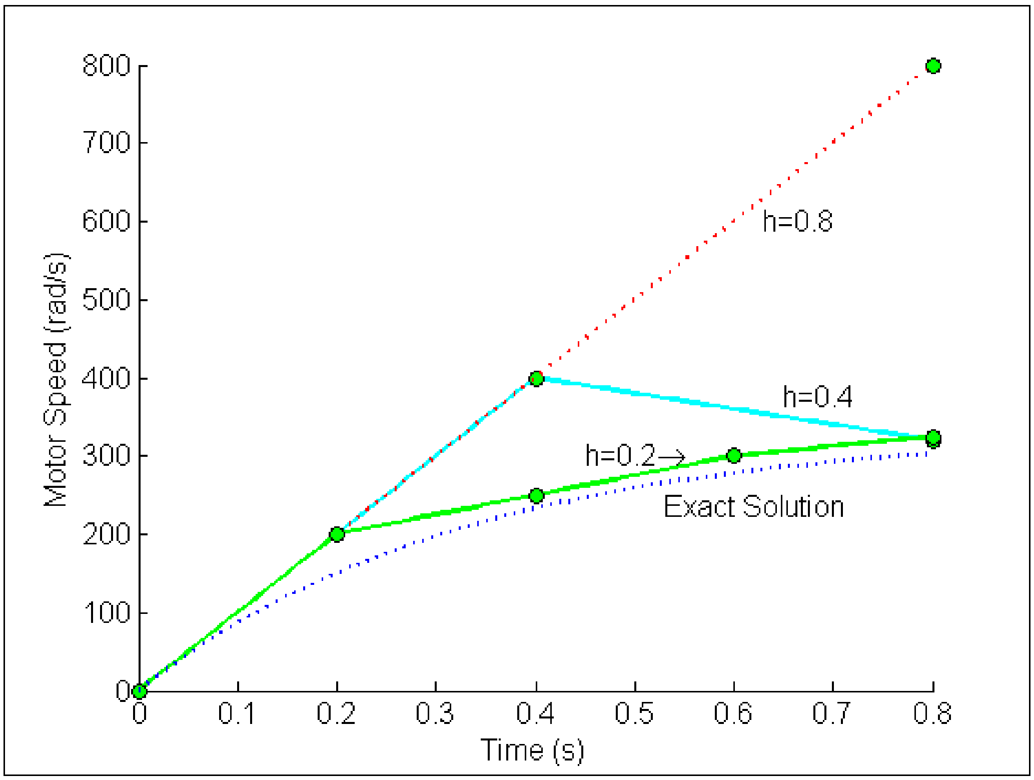

Figure 2 Comparison of Euler's method with exact solution for different step sizes.

The values of the calculated speed of the motor at $t = 0.8$ s as a function of step size are plotted in Figure 3.

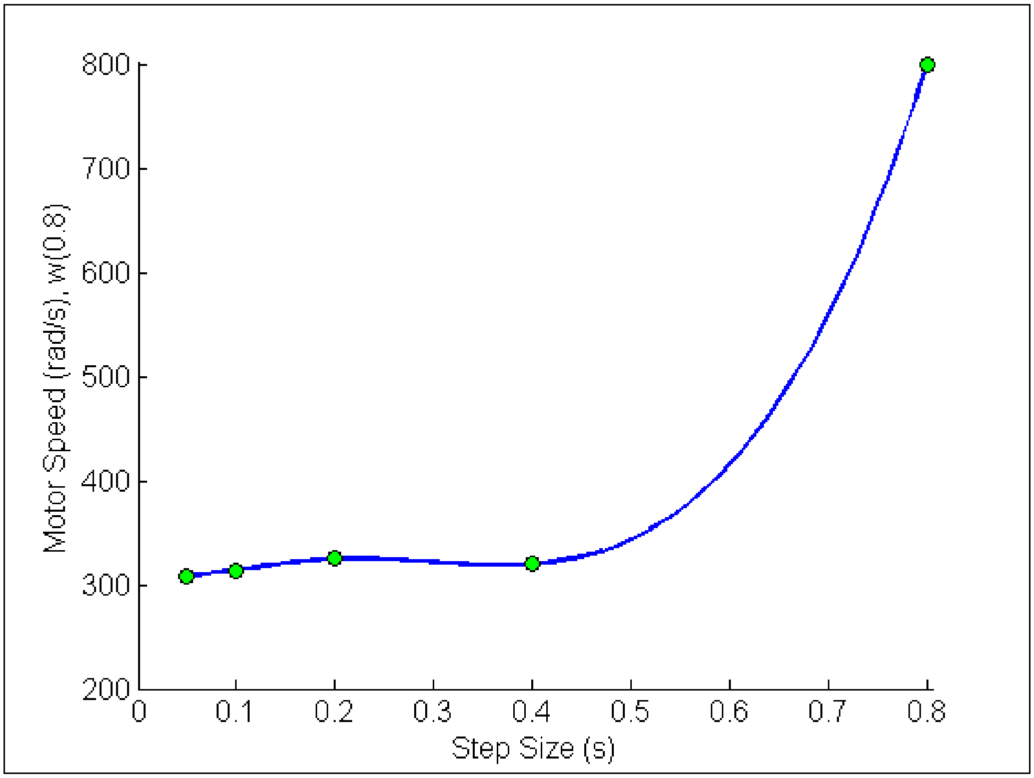

Figure 3 Effect of step size in Euler's method.

The exact solution of the ordinary differential equation is given by

$$w(t) = \left(\frac{1000}{3}\right) - \left(\frac{1000}{3}\right)e^{-3t}$$

The solution to this nonlinear equation at $t = 0.8\,\text{s}$ is

$w(0.8) = 303.09$ rad/s