List of rules of inference

This is a list of rules of inference, logical laws that relate to mathematical formulae.

Introduction

Rules of inference are syntactical **transform** rules which one can use to infer a conclusion from a premise to create an argument. A set of rules can be used to infer any valid conclusion if it is complete, while never inferring an invalid conclusion, if it is sound. A sound and complete set of rules need not include every rule in the following list, as many of the rules are redundant, and can be proven with the other rules.

Discharge rules permit inference from a subderivation based on a temporary assumption. Below, the notation

$$\varphi \vdash \psi$$

indicates such a subderivation from the temporary assumption φ to ψ .

Rules for classical sentential calculus

Sentential calculus is also known as propositional calculus.

Rules for negations

Reductio ad absurdum (or *Negation Introduction*)

$$\frac{\varphi \vdash \psi \quad \varphi \vdash \neg\psi}{\neg\varphi}$$

Reductio ad absurdum (related to the law of excluded middle)

$$\frac{\neg\varphi \vdash \psi \quad \neg\varphi \vdash \neg\psi}{\varphi}$$

Noncontradiction (or *Negation Elimination*)

$$\frac{\varphi \quad \neg\varphi}{\psi}$$

Double negation elimination

$$\frac{\neg\neg\varphi}{\varphi}$$

Double negation introduction

$$\frac{\varphi}{\neg\neg\varphi}$$

Rules for conditionals

Deduction theorem (or *Conditional Introduction*)

$$\frac{\varphi \vdash \psi}{\varphi \to \psi}$$

Modus ponens (or *Conditional Elimination*)

$$\frac{\varphi \to \psi \quad \varphi}{\psi}$$

Modus tollens

$$\frac{\varphi \to \psi \quad \neg\psi}{\neg\varphi}$$

Rules for conjunctions

Adjunction (or *Conjunction Introduction*)

$$\frac{\varphi \quad \psi}{\varphi \land \psi}$$

Simplification (or *Conjunction Elimination*)

$$\frac{\varphi \land \psi}{\varphi}$$

$$\frac{\varphi \land \psi}{\psi}$$

Rules for disjunctions

Addition (or *Disjunction Introduction*)

$$\frac{\varphi}{\varphi \lor \psi}$$

$$\frac{\psi}{\varphi \lor \psi}$$

Separation of Cases (or *Disjunction Elimination*)

$$\frac{\varphi \lor \psi \quad \varphi \to \chi \quad \psi \to \chi}{\chi}$$

Disjunctive syllogism

$$\frac{\varphi \lor \psi \quad \neg\varphi}{\psi}$$

$$\varphi \lor \psi$$

$$\frac{\neg\psi}{\varphi}$$

Rules for biconditionals

Biconditional introduction

$$\frac{\varphi \to \psi \quad \psi \to \varphi}{\varphi \leftrightarrow \psi}$$

Biconditional Elimination

$$\frac{\varphi \leftrightarrow \psi \quad \varphi}{\psi}$$

$$\frac{\varphi \leftrightarrow \psi \quad \psi}{\varphi}$$

$$\frac{\varphi \leftrightarrow \psi \quad \neg\varphi}{\neg\psi}$$

$$\frac{\varphi \leftrightarrow \psi \quad \neg\psi}{\neg\varphi}$$

$$\frac{\varphi \leftrightarrow \psi \quad \psi \vee \varphi}{\psi \wedge \varphi}$$

$$\frac{\varphi \leftrightarrow \psi \quad \neg\psi \vee \neg\varphi}{\neg\psi \wedge \neg\varphi}$$

Rules of classical predicate calculus

In the following rules, $\varphi(\beta/\alpha)$ is exactly like φ except for having the term β everywhere φ has the free variable α .

Universal Introduction (or *Universal Generalization*)

$$\frac{\varphi(\beta/\alpha)}{\forall\alpha\,\varphi}$$

Restriction 1: α does not occur in φ .

Restriction 2: β is not mentioned in any hypothesis or undischarged assumptions.

Universal Elimination (or *Universal Instantiation*)

$$\frac{\forall\alpha\,\varphi}{\varphi(\beta/\alpha)}$$

Restriction: No free occurrence of α in φ falls within the scope of a quantifier quantifying a variable occurring in β.

Existential Introduction (or *Existential Generalization*)

$$\frac{\varphi(\beta/\alpha)}{\exists\alpha\,\varphi}$$

Restriction: No free occurrence of α in φ falls within the scope of a quantifier quantifying a variable occurring in β.

Existential Elimination (or *Existential Instantiation*)

$$\frac{\begin{array}{l}\exists\alpha\,\varphi\\ \varphi(\beta/\alpha)\vdash\psi\end{array}}{\psi}$$

Restriction 1: No free occurrence of α in φ falls within the scope of a quantifier quantifying a variable occurring in β.

Restriction 2: There is no occurrence, free or bound, of β in ψ.

Table: Rules of Inference - a short summary

The rules above can be summed up in the following table.[1] The "Tautology" column shows how to interpret the notation of a given rule.

Rule of inference	Tautology	Name
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Simplification
p, q, $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
p, $p \rightarrow q$, $\therefore q$	$((p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$, $p \rightarrow q$, $\therefore \neg p$	$((\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$, $q \rightarrow r$, $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$, $\neg p$, $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
$p \vee q$, $\neg p \vee r$, $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

All rules use the basic logic operators. A complete table of "logic operators" is showed by a truth table, giving definitions of all the possible (16) truth functions of 2 boolean variables (p, q):

p	*q*	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
T	**T**	F	F	F	F	F	F	F	F	T	T	T	T	T	T	T	T
T	**F**	F	F	F	F	T	T	T	T	F	F	F	F	T	T	T	T
F	**T**	F	F	T	T	F	F	T	T	F	F	T	T	F	F	T	T
F	**F**	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T

where T = true and F = false, and, the columns are the logical operators: **0**, false, Contradiction; **1**, NOR, Logical NOR; **2**, Converse nonimplication; **3**, **¬p**, Negation; **4**, Material nonimplication; **5**, **¬q**, Negation; **6**, XOR, Exclusive disjunction; **7**, NAND, Logical NAND; **8**, AND, Logical conjunction; **9**, XNOR, If and only if, Logical biconditional; **10**, **q**, Projection function; **11**, if/then, Logical implication; **12**, **p**, Projection function; **13**, then/if, Converse implication; **14**, OR, Logical disjunction; **15**, true, Tautology.

Each logic operator can be used in a assertion about variables and operations, showing a basic rule of inference. Examples:

- The column-14 operator (OR), shows *Addition rule*: when *p*=T (the hypothesis selects the first two lines of the table), we see (at column-14) that *p*∨*q*=T.

  We can see also that, with the same premise, another conclusions are valid: columns 12, 14 and 15 are T.
- The column-8 operator (AND), shows *Simplification rule*: when *p*∧*q*=T (first line of the table), we see that *p*=T.

  With this premise, we also conclude that *q*=T, *p*∨*q*=T, etc. as showed by columns 9-15.
- The column-11 operator (IF/THEN), shows *Modus ponens rule*: when *p*→*q*=T and *p*=T only one line of the truth table (the first) satisfies these two conditions. On this line, *q* is also true. Therefore, whenever p → q is true and p is true, q must also be true.

Machines and well-trained people use this look at table approach to do basic inferences, and to check if other inferences (for the same premises) can be obtained.

Example 1

Let us consider the following assumptions: "If it rains today, then we will not go on a canoe today. If we do not go on a canoe trip today, then we will go on a canoe trip tomorrow. Therefore (Mathematical symbol for "therefore" is $\therefore$), if it rains today, we will go on a canoe trip tomorrow. To make use of the rules of inference in the above table we let p be the proposition "If it rains today", q be " We will not go on a canoe today" and let r be "We will go on a canoe trip tomorrow". Then this argument is of the form:

$$\begin{array}{l} \quad p \rightarrow q \\ \quad q \rightarrow r \\ \therefore \overline{p \rightarrow r} \end{array}$$

Example 2

Let us consider a more complex set of assumptions: "It is not sunny today and it is colder than yesterday". "We will go swimming only if it is sunny", "If we do not go swimming, then we will have a barbecue", and "If we will have a barbecue, then we will be home by sunset" lead to the conclusion "We will be home before sunset." Proof by rules of inference: Let p be the proposition "It is sunny this today", q the proposition "It is colder than yesterday", r the proposition "We will go swimming", s the proposition "We will have a barbecue", and t the proposition "We will be home by sunset". Then the hypotheses become $\neg p \wedge q, r \rightarrow p, \neg r \rightarrow s$ and $s \rightarrow t$. Using our intuition we conjecture that the conclusion might be t . Using the Rules of Inference table we can proof the conjecture easily:

Step	Reason
1. $\neg p \wedge q$	Hypothesis
2. $\neg p$	Simplification using Step 1
3. $r \rightarrow p$	Hypothesis
4. $\neg r$	Modus tollens using Step 2 and 3
5. $\neg r \rightarrow s$	Hypothesis
6. s	Modus ponens using Step 4 and 5
7. $s \rightarrow t$	Hypothesis
8. t	Modus ponens using Step 6 and 7

References

[1] Kenneth H. Rosen: *Discrete Mathematics and its Applications*,Fifth Edition, p. 58.

Article Sources and Contributors

List of rules of inference *Source*: http://en.wikipedia.org/w/index.php?oldid=433101960 *Contributors*: Anonymous Dissident, ArnoldReinhold, AugPi, Bkell, CBM, Colonies Chris, El C, Fireice, Gregbard, Infinity ive, JMRyan, JamesMazur22, Jitse Niesen, Jiy, Jonathanzung, Julian Mendez, Kumioko, LilHelpa, Magioladitis, Mark Renier, Mhss, Poor Yorick, Rezarob, Shadro, Sidmow, Stwalkerster, Toby Bartels, ZeroOne, 37 anonymous edits

License

Creative Commons Attribution-Share Alike 3.0 Unported
http://creativecommons.org/licenses/by-sa/3.0/