

$$= \begin{pmatrix} -6.7083 & 5.8506 & 5.2209 \\ 4.1472 \times 10^{-5} & 0.15476 & 1.1876 \\ -1.3916 \times 10^{-5} & -0.93681 & -0.44646 \end{pmatrix}$$

Clearly a real root is close to -6.7083 . Then the other roots can be obtained from the lower right block.

$$\begin{pmatrix} 0.15476 & 1.1876 \\ -0.93681 & -0.44646 \end{pmatrix}, \text{ eigenvalues: } -0.14585 + 1.011i, -0.14585 - 1.011i$$

How well do these work? Try the last one. Evaluating the polynomial at this value of x gives

$$6.3505 \times 10^{-4} + 2.0658 \times 10^{-4}i$$

11. Find the roots to $x^4 + 3x^3 + 4x^2 + x + 1$. It has two complex roots.

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{pmatrix}^{20} = \begin{pmatrix} 36482. & -27300.0 & -49899. & 2.4489 \times 10^5 \\ 14010.0 & 9182.0 & -77199. & 1.9499 \times 10^5 \\ 1.3180 \times 10^5 & -95190.0 & -1.9041 \times 10^5 & 9.0235 \times 10^5 \\ 27300.0 & 49899. & -2.4489 \times 10^5 & 5.4425 \times 10^5 \end{pmatrix} = \begin{pmatrix} 0.26030 & -6.9651 \times 10^{-2} & -0.94185 & 0.20079 \\ 9.9960 \times 10^{-2} & 0.25172 & 0.20932 & 0.93959 \\ 0.94038 & -0.20295 & 0.25314 & -0.10207 \\ 0.19478 & 0.94371 & -7.0906 \times 10^{-2} & -0.25775 \end{pmatrix} \begin{pmatrix} 1.4016 \times 10^5 & -85984. & -2.4746 \times 10^5 & 1.0378 \times 10^6 \\ 0 & 70622. & -2.0842 \times 10^5 & 3.6251 \times 10^5 \\ 0 & 0 & 2.1305 & -5.038 \\ 0 & 0 & 0 & 2.6834 \end{pmatrix}^T = \begin{pmatrix} 0.26030 & -6.9651 \times 10^{-2} & -0.94185 & 0.20079 \\ 9.9960 \times 10^{-2} & 0.25172 & 0.20932 & 0.93959 \\ 0.94038 & -0.20295 & 0.25314 & -0.10207 \\ 0.19478 & 0.94371 & -7.0906 \times 10^{-2} & -0.25775 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{pmatrix} = \begin{pmatrix} 0.26030 & -6.9651 \times 10^{-2} & -0.94185 & 0.20079 \\ 9.9960 \times 10^{-2} & 0.25172 & 0.20932 & 0.93959 \\ 0.94038 & -0.20295 & 0.25314 & -0.10207 \\ 0.19478 & 0.94371 & -7.0906 \times 10^{-2} & -0.25775 \end{pmatrix} = \begin{pmatrix} -0.61347 & -4.2510 & 0.48569 & 2.0968 \\ 0.50389 & -2.3376 & 0.11542 & 0.33094 \\ 2.5476 \times 10^{-7} & 8.4187 \times 10^{-6} & -0.15734 & 0.30446 \\ 4.6118 \times 10^{-6} & 7.5855 \times 10^{-6} & -0.97448 & 0.10846 \end{pmatrix}$$

Now there are two blocks of importance.

$$\begin{pmatrix} -0.61347 & -4.2510 \\ 0.50389 & -2.3376 \end{pmatrix}, \text{ eigenvalues: } -1.4755 + 1.1827i, -1.4755 - 1.1827i$$

$$\begin{pmatrix} -0.15734 & 0.30446 \\ -0.97448 & 0.10846 \end{pmatrix}, \text{ eigenvalues: } -0.02444 + 0.52823i, -0.02444 - 0.52823i$$

How well does one of these work? Try one.

$$\begin{aligned} &(-0.02444 + 0.52823i)^4 + 3(-0.02444 + 0.52823i)^3 \\ &+ 4(-0.02444 + 0.52823i)^2 + (-0.02444 + 0.52823i) \\ &+ 1 = 2.8879 \times 10^{-5} - 3.0092 \times 10^{-6}i \end{aligned}$$

12. Suppose A is a real symmetric matrix and the technique of reducing to an upper Hessenberg matrix is followed. Show the resulting upper Hessenberg matrix is actually equal to 0 on the top as well as the bottom.

Let $Q^T A Q = H$ where H is upper Hessenberg. Then take the transpose of both sides. This will show that $H = H^T$ and so H is zero on the top as well.