Do you remember Miguel? He had just finished working on figuring out the circumference of three different on deck pads for the pitchers to use while they warm up. Let’s look at his dilemma again before we look at the area of the pads.

Miguel’s latest task is to measure some different on deck pads for the pitchers to practice with. An on deck pad is a circular pad that is made up of a sponge and some fake grass. Pitchers practice their warm-ups by standing on them. They work on stretching and get ready to pitch the ball prior to their turn on the mound.

Miguel has three different on deck pads that he is working with. The coach has asked him to measure each one and find the circumference and the area of each.

Miguel knows that the circumference is the distance around the edge of the circle. He decides to start with figuring out the circumference of each circle.

He measures the distance across each one.

The first one measures 4 ft. across.

The second measures 5 ft. across.

The third one measures 6 ft across.

Miguel has completed the first part of this assignment. He knows the circumference of each pad. Now he has to figure out the area of each. Miguel isn’t sure how to do this. He can’t remember how to find the area of a circle. Miguel needs some help.

This is where you come in use this lesson to help you learn how to find the area of a circle. When finished, well come back to this problem and you can help Miguel figure out the area of each on deck pad.

What You Will Learn

By the end of this lesson, you will be able to demonstrate the following skills.
• Recognize the formula for the area of a circle.
• Find the area of circles given radius or diameter.
• Find radius or diameter given area.
• Find areas of combined figures involving parts of circles.

Teaching Time

I. Recognize the Formula for the Area of a Circle

In the last lesson you learned how to calculate the circumference of a circle. Let's take a few minutes to review the terms associated with circles.

A circle is a set of connected points equidistant from a center point. The diameter is the distance across the center of the circle and the radius is the distance from the center of the circle to the edge.

We also know that the number $\pi$, $\pi$, is the ratio of the diameter to the circumference. We use 3.14 to represent $\pi$ in operations.

What does all of this have to do with area?

Well, to find the area of a figure, we need to figure out the measurement of the space contained inside a two-dimensional figure. This is the measurement area. This is also the measurement inside a circle. You learned how to find the circumference of a circle, now let's look at using these parts to find the area of the circle.

How do we find the area of a circle?

The area of a circle is found by taking the measurement of the radius, squaring it and multiplying it by $\pi$.

Here is the formula.

$$A = \pi r^2$$

Write this formula down in your notebook.

II. Find the Area of Circles Given the Radius or Diameter

We already know that the symbol $\pi$ represents the number 3.14, so all we need to know to find the area of a circle is its radius. We simply put this number into the formula in place of $r$ and solve for the area, $A$.

Let's try it out.

Example

What is the area of the circle below?
We know that the radius of the circle is 12 centimeters. We put this number into the formula and solve for $A$.

$$A = \pi r^2$$
$$A = \pi (12)^2$$
$$A = 144 \pi$$
$$A = 452.16 \text{ cm}^2$$

*Remember that squaring a number is the same as multiplying it by itself.*

The area of a circle with a radius of 12 centimeters is 452.16 square centimeters.

Example

Some students have formed a circle to play dodge ball. The radius of the circle is 21 feet. What is the area of their dodge ball circle?
The dodge ball court forms a circle, so we can use the formula to find its area. We know that the radius of the circle is 21 feet, so let's put this into the formula and solve for area, $A$.

\[ A = \pi r^2 \]
\[ A = \pi (21)^2 \]
\[ A = 441 \pi \]
\[ A = 1384.74 \text{ ft}^2 \]

Notice that a circle with a larger radius of 21 feet has a much larger area: 1,384.74 square feet.

Sometimes, you will be given a problem with the diameter and not the radius. When this happens, you can divide the measurement of the diameter by two and then use the formula.

Example

Find the area of a circle with a diameter of 10 in.

First, we divide the measurement in half to find the radius.

\[ 10 \div 2 = 5 \text{ in} \]

Now we use the formula.

\[ A = \pi r^2 \]
\[ A = 3.14(5^2) \]
\[ A = 3.14(25) \]
\[ A = 78.5 \text{ square inches} \]

This is our answer.

9N. Lesson Exercises

Find the area of each circle.

1. Radius = 9 inches
2. Radius = 11 inches
3. Diameter = 8 ft.
Take a few minutes to check your work.

III. Find the Radius or Diameter Given the Area

We have seen that when we are given the radius or the diameter of a circle, we can find its area. We can also use the formula to find the radius or diameter if we know the area. Let’s see how this works.

Example

The area of a circle is 113.04 square inches. What is its radius?

This time we know the area and we need to find the radius. We can put the number for area into the formula and use it to solve for the radius, \( r \).

\[
A = \pi r^2 \\
113.04 = \pi r^2 \\
113.04 \div \pi = r^2 \\
36 = r^2 \\
\sqrt{36} = r \\
6 \text{ in.} = r
\]

Let’s look at what we did to solve this. To solve this problem we needed to isolate the variable \( r \). First, we divided both sides by \( \pi \), or 3.14. Then, to remove the exponent, we took the square root of both sides. A square root is a number that, when multiplied by itself, gives the number shown. We know that 6 is the square root of 36 because \( 6 \times 6 = 36 \).

The radius of a circle with an area of 113.04 square inches is 6 inches.

Example

What is the diameter of a circle whose area is 379.94 \( cm^2 \)?

What is this problem asking us to find? We need to find the diameter (not the radius!). What information is given in the problem? We know the area. Therefore we can use the formula to solve for the radius, \( r \). Once we know the radius, we can find the diameter. Let’s give it a try.

\[
A = \pi r^2 \\
379.94 = \pi r^2 \\
379.94 \div \pi = r^2 \\
121 = r^2 \\
\sqrt{121} = r \\
11 \text{ cm} = r
\]

The radius of a circle with an area of 379.94 square centimeters is 11 centimeters.
Remember, this problem asked us to find the diameter, so were not done yet. How can we find the diameter? The diameter is always twice the length of the radius, so the diameter of this circle is $11 \times 2 = 22$ centimeters.

As we have seen, we can use the area formula whenever we are given information about a circle. If we know the diameter or radius, we can solve for the area, $A$. If we are given the area, we can solve for the radius, $r$. If we know the radius, we can also find the diameter.

9O. Lesson Exercises

Find the radius of each circle.

1. Area = 153.86 sq. in.
2. Area = 354.34 sq. ft.
3. 452.16 sq. m

Check your work with a partner and then continue with the next section.

IV. Find Areas of Combined Figures Involving Parts of Circles

Sometimes we may be asked to find the area of a combined figure. Combined figures often include portions of circles, such as a quarter or semicircle (which is a half circle). We can find the area of combined figures by breaking them down into smaller shapes and finding the area of each piece.

We can calculate the area of a portion of a circle. As long as we know the radius of the circle, we can find its area. Then we can divide that area into smaller pieces or subtract a portion to find the area of part of the circle. Let's try this out.

Example

What is the area of the figure below?

This figure is a semicircle, or half of a circle. Remember that a diameter always divides a circle in half. Therefore the edge measuring 17 inches is the circles diameter. Can we use it to find the area of the whole circle?

We sure can! The radius of the circle must be $17 \div 2 = 8.5$ inches. Now let's use the formula to solve for area.
We know that a whole circle with a radius of 8.5 inches (and a diameter of 17 inches) is 226.87 square inches. Therefore the semicircle figure has an area of $226.87 \div 2 = 113.44$ square inches. As long as we can find the area of a whole circle, we can divide or subtract to find the area of a portion of a circle.

Now lets look at a combined figure.

Example

Find the area of the figure below.

First, we have to find the area of the rectangle. We can do this by multiplying the length times the width. Then we can find the area of the circle. If you notice, the width of the rectangle is also the diameter of the circle. This will help us when we want to find the area of the circle.

Lets start with the rectangle.

\[
A = lw
\]
\[
A = 6(8)
\]
\[
A = 48
\]

The area is 48 square inches for the rectangle.

Now lets look at the semi-circle. If the diameter is the width which is 6 inches, then the radius is 3 inches. We can find the area of a circle now.
\[
A = \pi r^2 \\
A = 3.14(3^2) \\
A = 3.14(9) \\
A = 28.26 \text{ sq. inches}
\]

Now this is the area of a whole circle. We only need the area of a semi-circle. Let's divide this value in half.

The area of the semi-circle is 14.13 square inches.

**Now we add the two areas together.**

\[48 + 14.13 = 62.13 \text{ square inches}\]

The area of the entire figure is 62.13 square inches.

Now let's go and use what we have learned to help Miguel figure out the area of each on-deck pad.

**RealLife Example Completed**

**The On-Deck Pads**

![Image of a child practicing baseball](image)

Here is the original problem once again. Reread it and underline any important information.

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Miguel has completed the first part of this assignment. He knows the circumference of each pad. Now he has to figure out the area of each. Miguel isn’t sure how to do this. He can’t remember how to find the area of a circle. Miguel needs some help.

Now it’s time to help Miguel figure out each area.

The first one has a diameter of 4 feet, so it has a radius of 2 ft. Here is the area of the first pad.

\[
A = \pi r^2 \\
A = 3.14(2^2) \\
A = 12.56 \text{ sq. feet}
\]

The second pad has a diameter of 5 feet, so it has a radius of 2.5 feet.

\[
A = \pi r^2 \\
A = 3.14(2.5^2) \\
A = 19.63 \text{ sq. feet}
\]

The third pad has a diameter of 6 feet, so it has a radius of 3 feet.

\[
A = \pi r^2 \\
A = 3.14(3^2) \\
A = 28.26 \text{ sq. feet}
\]

Miguel is very pleased with his work. He is sure that his coach will be pleased with his efforts as well!

**Vocabulary**

Here are the vocabulary words that are found in this lesson.

**Circle**

a set of connected points that are equidistant from a center point.

**Diameter**

the distance across the center of a circle.

**Radius**

the distance from the center of the circle to the outer edge.

**Area**

the space inside a two-dimensional figure
Technology Integration

1. [http://www.mathplayground.com/mv_area_circles.html](http://www.mathplayground.com/mv_area_circles.html) This is a Brightstorm video on finding the area of a circle.

Time to Practice

Directions: Find the area of each circle given the radius or diameter.

1. \( r = 3 \text{ in} \)
2. \( r = 5 \text{ in} \)
3. \( r = 4 \text{ ft} \)
4. \( r = 7 \text{ m} \)
5. \( r = 6 \text{ cm} \)
6. \( r = 3.5 \text{ in} \)
7. \( d = 16 \text{ in} \)
8. \( d = 14 \text{ cm} \)
9. \( d = 20 \text{ in} \)
10. \( d = 15 \text{ m} \)
11. \( d = 22 \text{ cm} \)
12. \( d = 24 \text{ mm} \)

Directions: Find each radius given the area of the circle.

13. 12.56 sq. in.
14. 78.5 sq. m
15. 200.96 sq. cm
16. 254.34 sq. in
17. 7.07 sq. ft.
18. 28.26 sq. m

Directions: Find each diameter given the area of the circle.

19. 12.56 sq. in.
20. 78.5 sq. m
21. 200.96 sq. cm
22. 254.34 sq. in
23. 7.07 sq. ft.
24. 28.26 sq. m

Directions: Solve each problem.

25. Rob is painting large polka dots on a sheet for the backdrop of the school musical. He painted 16 polka dots, each with a radius of 3 feet. What is the total area that the polka dots cover?

26. The librarian is having the library at her school carpeted. The library is a circular room with a diameter of 420 feet. How many square feet of carpet will she need to order?