
Students are able to extend their knowledge of ratios and proportions to applications in real life situations. These include finding tax, tip, percent increase and decrease, commission, and simple interest.

Percents are used more than just as grades in school - percents give us a way to see the comparisons of two things (like a ratio) in a common way that people can understand. If I said that you were 85% done with your homework, you would know that you were almost done. If I said that you had to put 20% of your allowance away for college, you would know that it was less than half, and you would still have most of your money left. In this concept, we will learn some of the main uses of percents in daily life.

Tax and Gratuities

Taxes are monies that are collected by the government to help pay for services such as schools, libraries, roads, and police and fire protection. A sales tax is a tax on something bought in a store. The rate of sales tax is given as a percent. A percent can be written as a ratio out of 100. You can find the amount of sales tax by using the following proportion:

\[
\frac{\text{percent sales tax}}{100} = \frac{\text{amount of sales tax}}{\text{amount to be taxed}}
\]

Let's say you bought a jacket for $85. If the sales tax is 7.5%, what is the tax? What would be the total cost of the jacket? Remember that we can only add percents to percents. A percent must be changed to a rational number to be added to another rational number.

First we find the amount of sales tax.

\[
\frac{\text{percent of sales tax}}{100} = \frac{\text{amount of sales tax}}{\text{amount to be taxed}}
\]

\[
\frac{7.5}{100} = \frac{t}{85}
\]

\[
(7.5)(85) = (100)(t)
\]

\[
637.5 = 100t
\]

\[
\frac{637.5}{100} = \frac{100t}{100}
\]

\[
6.375 = t
\]
You need to round $6.375 to the nearest penny, which is $6.38.

Add the price of the jacket and the sales tax.

$85 + $6.38 = $91.38

The total cost of the jacket is $91.38.

What if you were only interested in finding the final price? There is a little trick to this. We can eliminate a step. What you have to understand is that the price to be taxed is 100% - you are taxing the whole amount!

So if you are paying an 8% sales tax, then you are really paying 108% of the cost.

\[
\text{price of the item(s)} + \text{amount of sales tax} = \text{total to be paid} \\
100\% + 8\% = 108\%
\]

This changes our proportion to include both the price and sales tax all at once.

\[
\frac{\text{total percent to pay}}{100} = \frac{\text{total amount to pay}}{\text{amount to be taxed}}
\]

Try this method with the following example.

**Example 1**

Reese is buying a smart TV. It will cost $1,450. The tax rate is 8.5%. What is the total amount that Reese will pay, including tax?

*First we find the numbers to put in the proportion.*

- Total percent to pay → 100% + 8.5% = 108.5%
- Amount to be taxed → $1,450
- Total amount to pay → ...we don’t know this so we use a variable. Let the total amount to pay = $t$.

*Now we can set up one proportion that will find the total to be paid.*

\[
\frac{\text{total percent to pay}}{100} = \frac{\text{total amount to pay}}{\text{amount to be taxed}} \\
\frac{108.5}{100} = \frac{t}{1450} \\
(108.5)(1450) = (100)(t) \\
157325 = 100t \\
\frac{157325}{100} = \frac{100t}{100} \\
1573.25 = t
\]

*Reese would pay $1,573.25 as his total.*
Gratuities is a fancy word for a tip that you leave for people who do a good job for you. People that get a tip could be a waiter, a hairdresser, a person who cares for your child or pet, or even the person who delivers your paper. You find the tip using a proportion just like the one for finding sales tax. A tip is usually found separate from the total because a tip is usually given separately from the cost of the service. The percent of a tip is usually decided by the person giving the tip.

Here is the proportion for finding the gratuity (tip). The percent always goes over the 100, and the actual cost always goes in the denominator of the second ratio.

\[
\frac{\text{percent of tip}}{100} = \frac{\text{amount of tip}}{\text{amount of cost of the service}}
\]

If three of your friends go with you to a restaurant and the meal costs $53.38, how much should you leave as a tip? The usual tip at a restaurant is 15% of the cost of the meal. You can find this out by using the proportion for finding the amount of tip.

\[
\frac{15}{100} = \frac{t}{53.38}
\]

\[
800.7 = 100t
\]

\[
8.007 = t
\]

We have to round any money to the nearest penny. So 8.007 becomes $8.01. The tip that you and your friends should leave is $8.01.

Now here is an example that combines both tax and tip. It is very important to know that the tip is on the meal without the tax added in. Tax and tip are each on the meal alone.

**Example 2**

Taylor and her Dad went out to a restaurant for dinner. It had been a long day at the candy store that they owned, and they were both hungry. The cost of the meal was $25.75. Taylor and her Dad needed to figure out the total bill given a 7% sales tax and a 15% tip.

Using proportions, find the total bill including tax and tip. Show all work and label your final answers.

*First find the tax on the cost of the meal.*

\[
\frac{\text{percent tax}}{100} = \frac{\text{amount of tax}}{\text{amount to be taxed}}
\]

• Percent tax is 7
• Amount to be taxed is $25.75
• Amount of tax is not known so we choose a variable. Let the amount of tax = x

Now replace with the numbers or variable and solve the proportion.

\[
\frac{7}{100} = \frac{x}{25.75} \\
(7)(25.75) = (100)(x) \\
180.25 = 100x \\
\frac{180.25}{100} = \frac{100x}{100} \\
1.8025 = x
\]

Rounding to the nearest penny, the tax is $1.80

Next find the tip on the cost of the meal.

\[
\frac{\text{percent tip}}{100} = \frac{\text{amount of tip}}{\text{amount of cost of the service}}
\]

Percent tip is 15
Amount tip is based on is $25.75
Amount of tip is not known so we choose a variable. Let the amount of the tip = t.

Now replace with the numbers or variable and solve the proportion.

\[
\frac{15}{100} = \frac{t}{25.75} \\
(15)(25.75) = (100)(t) \\
386.25 = 100t \\
\frac{386.25}{100} = \frac{100t}{100} \\
3.8625 = t
\]

Rounding to the nearest penny, the tip is $3.86.

Finally add the cost of the meal, the tax, and the tip together. This is the total that Taylor and her dad will pay.

\[
25.75 + 1.80 + 3.86 = 31.41
\]

The total to pay is $31.41.

[http://www.funbrain.com/penguin/index.html](http://www.funbrain.com/penguin/index.html) is a game that lets you practice calculating tip.

Salespeople often earn a commission. A commission is an amount of money based on how much they sell. So the more you sell, the bigger your commission. Most companies that pay commission do it based on a percentage of the number of sales. Confused? Look at the proportion for finding the tax and the tip. Then look at the proportion for finding commission. Do you notice the pattern in all three?

Real estate agents make their living on commission. The rate of commission for selling a house is 6% of the cost of the house. If a real estate agent sold a house for $128,000 how much would the commission be?

Use the proportion for commission.

We know:
- the percent commission is 6
- the amount of sale is $128,000
- the amount of commission...we don’t know. So we use a variable. Let the amount of commission = \( c \).

\[
\frac{\text{percent commission}}{100} = \frac{\text{amount of commission}}{\text{amount of sale}}
\]

\[
\frac{6}{100} = \frac{c}{128,000}
\]

\[
(6) (128,000) = (100) (c)
\]

\[
768,000 = 100c
\]

\[
\frac{768,000}{100} = \frac{100c}{100}
\]

\[
7,680 = c
\]

The amount of commission the real estate agent would make is $7,680.

We can also use the same proportion to find the percent of commission (commission rate). The example below shows how.

**Example 3**

Robyn earned $150.00 as a commission selling a painting. The painting was sold for $2,550.00. What is the rate of her commission? Round to the nearest tenth of a percent if needed.

*First we decide what we know and what we do not know.*
• The amount of commission is 150
• The amount of the sale is 2,550
• The percent of commission...we do not know. We use a variable. Let the percent of commission = p

Put the information into the proportion for commission and solve for the variable p.

\[
\frac{\text{percent of commission}}{100} = \frac{\text{amount of commission}}{\text{amount of sale}}
\]

\[
\frac{p}{100} = \frac{150}{2550}
\]

\[
(p)(2550) = (100)(150)
\]

\[
2550p = 15000
\]

\[
\frac{2550p}{2550} = \frac{15000}{2550}
\]

\[
p = 5.88235941
\]

Rounding to the nearest tenth would be 5.9% commission.

The site http://www.aaamath.com/rat68_x3.htm#pgtp has more practice on finding commission

**Percent of Increase and Percent of Decrease**

Prices can increase. Costs can increase. Numbers can increase. We can find the percent of increase when dealing with an increase. All of these things can also decrease. When there has been an increase or decrease from an original amount to a new amount, we can find the percent of increase or decrease using the same basic steps. Since we use the same steps for both, we simply call it "finding the percent of change".

How do we find the percent of change?

The percent of change from one amount to another is the ratio of the amount of change to the original amount. This is the proportion for finding percent of change.

\[
\frac{\text{amount of change}}{\text{original amount}} = \frac{\text{percent of change}}{100}
\]

To find the amount of change just subtract the original price and the new price. The original price is the starting price and the new price is the ending price. 100 is always the denominator because a percent is always out of 100.

If the original price of an item is $58, and is now priced at $42, what is the percent of change to the nearest whole number?

The two prices are $58 and $42. To find the amount of change, just subtract the two.

\[
58 - 42 = 16
\]

This is the amount of change. Now we find the numbers to put in the proportion.

• Amount of change is 16
• Original amount is 58
• Percent of change is ...we don’t know this so we use a variable. Let the percent of change = \( p \)

\[
\frac{\text{amount of change}}{\text{original amount}} = \frac{\text{percent of change}}{100}
\]

\[
\frac{16}{58} = \frac{p}{100}
\]

\[
(16)(100) = (58)(p)
\]

\[
1600 = 58p
\]

\[
\frac{1600}{58} = \frac{58p}{58}
\]

\[
27.5862069 = p
\]

Now we round our answer to the nearest whole number. 27.5862069 becomes 28%.

Is the 28% showing an increase in the price or a decrease in the price? The price is going down so the 28% is called a percent markdown.

**Example 4**

Find the percent markup of an item that was purchased at $35 and put on a shelf to sell for $55. Round to the nearest whole number if needed.

*First we find the amount of change. 55 - 35 = 20. The amount of change is 20.*

*Now we list all the parts of the proportion.*

• Amount of change is 20
• Original amount is 35
• Percent of change is ....we don’t know so we use a variable. Let the percent of change = \( p \).

*Put all parts in the proportion. Solve for \( p \).*

\[
\frac{\text{amount of change}}{\text{original amount}} = \frac{\text{percent of change}}{100}
\]

\[
\frac{20}{35} = \frac{p}{100}
\]

\[
(20)(100) = (35)(p)
\]

\[
2000 = 35p
\]

\[
\frac{2000}{35} = \frac{35p}{35}
\]

\[
57.14285714 = p
\]

*Round to the nearest whole number.*

*The percent markup is 57%.*
Example 5

Find the percent change of an item that started at $166.50 and ended at $123.50. Round to the nearest whole number if needed. Indicate if this is a percent increase or decrease.

First find the amount of change. Subtract the two prices.

\[ 166.50 - 123.50 = 43 \]

Now put all the information into the proportion and solve for the percent. Always state what the variable stands for.

Let the percent change = \( p \)

\[
\frac{\text{amount of change}}{\text{original amount}} = \frac{\text{percent of change}}{100}
\]

\[
\frac{43}{166.50} = \frac{p}{100}
\]

\[
(43)(100) = (166.50)(p)
\]

\[
4300 = 166.50p
\]

\[
\frac{4300}{166.50} = \frac{166.50p}{166.50}
\]

\[
25.825825\ldots = p
\]

Rounding to the nearest whole number gives 26%. The price is going down so it is a percent decrease or percent markdown.

Example 6

Rosie bought some tickets to a concert and found out that she could not go. She decided to sell them online. Rosie wanted to markup the price 7% to make some money. If she bought the tickets for $80, what does she need to sell them for a 7% markup? If the site charges a fee of $2.00 to list the tickets, what money will Rosie get from selling the tickets online?

Markup is just like percent increase. Markup means that the price is 100% plus the 7% she wants to make. The new price of the tickets will be 107% of the price ($80). We use the same proportion as for percent increase.

What we know and do not know:

- The percent markup will be 107% (this is just like the percent of change).
- The original amount is $80.
- The new price of the tickets on line...we do not know. So we use a variable. Let the price on line = \( p \).

Now put all the information into the proportion and solve.
The price on line would be $85.60.

If the site charges $2.00 to post the tickets, subtract $2.00 from $85.60.

\[ 85.60 - 2.00 = 83.60 \]

If Rosie bought the tickets for $80 and she gets $83.60 after selling them on line, she will make a profit of $3.60.

http://www.youtube.com/watch?v=CLbRHY1YPK4 is a demonstration you can watch on how to find percent increase and decrease.

Additional practice problems with hints and answers can be found at http://www.algebralab.org/Word/Word.aspx?file=Algebra_PercentsII.xml

http://www.onlinemathlearning.com/percent-increase-decrease.html has two videos that you can watch on percent increase and decrease.

Percent of Error

Sam came back from his run and his friend asked how far he ran. Sam figured about 3.5 miles. Later they drove the course and found out it was actually 4.1 miles. How precise was his guess? How close is "close enough"?

There is a way to figure out, using math, how good someone’s guess, measure, or prediction is compared to the actual number. It is by finding the relative error and the percent of error.
• **Precision** - how close the measurement is to the exact measure
• **Error** - the difference between the exact value and the "about" measure
• **Relative error** - \( \frac{\text{error}}{\text{exact value}} \)
• **Percent of error** - relative error written as a percent

So let’s get back to Sam’s run. We want to see how precise his measurement is. Finding this is like finding percent of change. First there is subtraction, then a proportion.

Step one is to find the error. This means to subtract the two measures. \( 4.1 - 3.5 = .6 \).

Now we set up a proportion to find the percent of error. Here is the general set-up.

\[
\frac{\text{percent of error}}{100} = \frac{\text{error}}{\text{exact value}}
\]

Now we decide what we do and don’t know to set up the proportion for Sam’s run.

- **Error** - the error is .6
- **Exact value** - the exact value is 4.1
- **Percent of error** - we do not know, so we use a variable. Let the percent of error = \( p \).

Now put all the information in the proportion and solve.

\[
\frac{p}{100} = \frac{.6}{4.1}
\]

\[
(p)(4.1) = (100)(.6)
\]

\[
4.1p = 60
\]

\[
\frac{4.1p}{4.1} = \frac{60}{4.1}
\]

\[
p = 14.63414634... or 14\frac{26}{41}
\]

(because the decimal repeats, we can change it to a fraction to keep the answer exact)

It seems that Sam’s approximation of how far he ran is over 14% off! (14\(\frac{26}{41}\)% to be exact.)

You can make a game out of percent error. Have you and some friends each guess how many students are in your entire grade or even in your entire school. Then find out from your teacher how many are actually in your grade or school. Use the two steps to see who had the least percent of error. The least percent of error is the closest to the actual amount.

Percent of error is also used in measuring lengths. If you measure using a ruler marked with eighths and a friend has a ruler marked with sixteenths, who would have the more accurate measure? Most likely your friend. By using sixteenths, his measure can be more precise. Here is an example of how ruler measurements relate to percent error.

**Example 7**

George measured the diagonal of a rectangle and found it to be \( 16\frac{3}{4} \) inches. The actual measurement of the diagonal (calculated using the Pythagorean Theorem) found the diagonal to be 17 inches. What is the percent error of George’s measurement?
First find the error by subtracting the actual and the measured lengths.

\[ 17 - 16\frac{3}{4} = \frac{1}{4} \]

Then list the parts of the proportion.

- Error is \( \frac{1}{4} \) inch.
- Actual measure is 17 inches.
- Percent of error is not known. Let the percent of error = \( p \).

Put all the information in the proportion and solve.

\[
\frac{\text{percent of error}}{100} = \frac{\text{error}}{\text{exact value}}
\]

\[
\frac{p}{100} = \frac{\frac{1}{4}}{17}
\]

\[
(p)(17) = (100) \left( \frac{1}{4} \right)
\]

\[
17p = 25
\]

\[
\frac{17p}{17} = \frac{25}{17}
\]

\[
p = 1.470588235 \text{ or } 1 \frac{8}{17}
\]

This means that George’s percent of error is \( 1 \frac{8}{17} \% \) off. This looks like a pretty good measurement.

**Simple Interest**

Interest is the extra money you earn when you put money into a savings account, stock, or investment fund. Interest can also be the extra money you have to pay back to the bank when you borrow money, for a car or house. In simple terms, if you put money in - you get interest. If you take money out - you pay interest.

- **Interest** - the extra money you get or have to pay back
- **Principal** - the original amount of money you invest or borrow
- **Rate** - the percent that is earned or charged on your money
- **Time** - the number of years (or part of a year) you invest or borrow the money

**Simple interest** is calculated using time in years. If you are given a problem to find simple interest and the time is in months, the chart below shows months changed into years. To change months into years - divide the months by 12 since there are 12 months in a year.
Always use years or part of a year for the time in the formula.

A percentage rate is really a unit rate. It is the amount charged for each dollar that you invest or borrow. A rate of 3% is really three cents for every dollar. We can write this as the unit rate $\frac{0.03}{1}$.

Let’s say you borrowed $500.00 from the bank. The bank charges 5% interest annually. (Annually means for 1 year.) If you pay back all the money at the end of 2 years, how much interest will you pay?

To figure this out, let’s use our unit rate and make an equation using equivalent proportions. The rate is 5%. This means that the unit rate is $\frac{0.05}{1}$ (5 cents for every dollar). We want to know how much annual interest for $500.

\[
\frac{0.05}{1} = \frac{i}{500}
\]

\[(500)(0.05) = (1)(i)\]

\[25 = i\]

This means for 1 year, you have to pay an extra $25 to the bank. But you want to pay it back in two years. So we just multiply our interest for 1 year by 2.

\[25 \times 2 = 50\]

You would have to pay an extra $50 back to the bank.

What if it had been 3 years before you had paid back the money? If this was the case, we would just multiply our interest per year by 3. We could even do this by using another proportion.

\[
\frac{25(\text{dollars})}{1(\text{year})} = \frac{i}{3(\text{years})}
\]

\[(25)(3) = (1)(i)\]

\[75 = i\]

You would have paid $75.00 in interest.

Now we have to remember that the interest is the extra we have to pay back. If we want to know the total amount to pay back, we add the interest to the amount we borrow. If we are investing money, the interest is added on to the money you invested.

Using our example of borrowing $500 from the bank for two years, we found that the interest was $50. To find the total to pay back, add the principal (the amount borrowed) and the interest.

\[
\text{principal} + \text{interest} = \text{total to pay back}
\]

\[500 + 50 = \text{total to pay back}\]

\[550 = \text{total to pay back}\]

$550 must be paid back to the bank.

Now we will go in the reverse direction. We will start off with the total to be paid back and find the interest rate.

Example 8
If you borrow $3,600 from USA Savings Bank for 18 months, at the end of the 18 months you will repay $4,059 to the bank. What is the interest rate for this loan?

*The amount to be repaid includes the principal plus the interest. Use the same equation as the model and solve for the interest. Remember that this interest is the money NOT the interest rate. Always define the variable.*

*Let the interest = i*

\[ \text{principal} + \text{interest} = \text{amount to pay back} \]
\[ 3600 + i = 4059 \]
\[ 3600 - 3600 + i = 4059 - 3600 \]
\[ i = 459 \]

The interest paid back is $459. Now we find the interest for 1 year. Since there are 12 months in a year, 18 months is \( \frac{18}{12} \) or 1.5 years. We make a proportion and solve for the interest for 1 year (the unit rate for interest). Always define the variable.

*Let the interest for one year = x.*

\[ \frac{459(\text{interest})}{1.5(\text{years})} = \frac{x(\text{interest})}{1(\text{year})} \]
\[ (459)(1) = (1.5)(x) \]
\[ 459 = 1.5x \]
\[ 459 \quad 1.5x \]
\[ 1.5 \quad 1.5 \]
\[ 306 = x \]

This is the interest for one year. Still not done – one more proportion to go!

Now that we know the interest for one year, we can find the interest rate for 1 year. Just set up another proportion. Always define the variable.

*Let the interest rate = r.*

\[ \frac{306(\text{interest})}{3600(\text{dollars})} = \frac{r(\text{interest})}{1(\text{dollars})} \]
\[ (306)(1) = (3600)(r) \]
\[ 306 = 3600r \]
\[ 306 \quad 306r \]
\[ 3600 \quad 306 \]
\[ .085 = r \]

This is the amount of interest for $1. It is the unit rate.

Now we make .085 into a percent.
\[ .085 \times 100 = 8.5\% \]

Finally! The interest rate is 8.5%.
Example 9

Derek is going to buy his first car. He has to borrow $8,000. He needs to find the best car loan possible. Here are his two choices:

![Greater Bank logo]

Derek wants the loan with the least amount of interest. Which loan should he select? Justify your answer using simple interest.

*Find the amount of interest on the first option.* 3% means 3 cents extra for every dollar borrowed. The unit rate is \( \frac{0.03}{1} \). Set up a proportion to find the amount of interest for $8,000 for 1 year. We set our interest = \( i \).

\[
\frac{0.03}{1} = \frac{i}{8000} \\
(0.03)(8000) = (1)(i) \\
240 = i
\]

This is the interest for one year. The option is for 5 years. Multiply 240 by 5 to get the total interest to be paid back.

\[240 \times 5 = 1200\]

Derek will pay $1,200 in interest if he takes the first option.

Next find the amount of interest on the second option. 7% means 7 cents extra for every dollar borrowed. The unit rate is \( \frac{0.07}{1} \). Set up a proportion to find the amount of interest for $8,000 for 1 year. We set our interest = \( i \).

\[
\frac{0.07}{1} = \frac{i}{8000} \\
(0.07)(8000) = (1)(i) \\
560 = i
\]

This is the interest for one year. The option is for 2 years. Multiply 560 by 2 to get the total interest to be paid back.

\[560 \times 2 = 1,120\]

Derek will pay $1,120 in interest if he takes the second option.

The option with the least amount of interest to be paid back is option 2.
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