Surface Area of Rectangular Prisms

Jen Kershaw
Here you’ll learn to find the surface area of rectangular prisms using formulas.

On her first day wrapping boxes, Candice had a customer come up to the counter with a huge box in her arms.

“This is a doll house for my niece,” the woman said. “Can you please wrap it?”

“Certainly,” Candice said looking at the box.

The box was $40'' L \times 23'' W \times 32'' H$. Candice looked at the different rolls of wrapping paper. She was sure that she was going to need the largest roll of wrapping paper that she could find, and even then she was sure that it was going to take two pieces of paper. The wrapping station has a huge roll of paper on a big roller. This one was brand new.

“How big is this roll?” Candice asked Mrs. Scott.

“It is 24'' W × 900 feet,” Mrs. Scott said.

“That should be big enough,” Candice thought.

Candice isn’t sure that the roll of paper will be enough for the doll house. This is a problem that can be solved by surface area. If Candice can figure out the surface area of the box, then she will know how much of the wrapping paper she will need to wrap the doll house.

This Concept will teach you all about surface area. Pay attention and at the end of the Concept you will know how to figure out the wrapping paper dilemma.

**Guidance**

In this Concept, we will look at prisms with more detail. Remember that a **prism** is a three-dimensional object with two congruent parallel bases. The shape of the base names the prism and there are rectangles for the sides of the prism.

When we worked with two-dimensional figures, we measured the area of those figures. **The area is the space that is contained in a two-dimensional figure.** Now we are going to look at the area of three-dimensional figures. Only this isn’t called simply area anymore, it is called surface area.
What is surface area?

The **surface area** is the covering of a three-dimensional figure. Imagine you could wrap one of the figures above in wrapping paper, like a present. The amount of wrapping paper needed to cover the figure represents its surface area.

To find the surface area, we must be able to calculate the area of each face and then add these areas together. One way to do this is to use a net. Remember that a net is a two-dimensional representation of a three-dimensional solid. A net is a stretched out picture or an unfolded picture of a solid.

If we look at a net and find the sum of each surface of the net and then add up each measurement, then we will know the measurement of the “cover” of the figure.

We can do this with prisms of all different kinds. Let’s look at a net for a rectangular prism.

Now we can find the area of each part of the rectangular prism. Notice that the rectangular prism is made up of rectangles. To find the area of a rectangle, we use this formula.

\[ A = lw \]

Next we can find the area of each part of the prism. Remember that there are six faces that we need to measure!

<table>
<thead>
<tr>
<th>Bottom face</th>
<th>top face</th>
<th>long side</th>
<th>long side</th>
<th>short side</th>
<th>short side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = lw )</td>
<td>( A = lw )</td>
<td>( A = lh )</td>
<td>( A = lh )</td>
<td>( A = wh )</td>
<td>( A = wh )</td>
</tr>
<tr>
<td>( 12 \times 7 )</td>
<td>( 12 \times 7 )</td>
<td>( 12 \times 3 )</td>
<td>( 12 \times 3 )</td>
<td>( 7 \times 3 )</td>
<td>( 7 \times 3 )</td>
</tr>
<tr>
<td>84</td>
<td>84</td>
<td>36</td>
<td>36</td>
<td>21</td>
<td>21</td>
</tr>
</tbody>
</table>

The answer is 282 sq. inches.

Notice that because surface area involves “area” that we use square units to label each measurement!!

We found the area of each rectangular face and then added all of these areas together. The total surface area of the rectangular prism is 282 square inches. Using a net helped us to locate all of the faces and find the measurements of each side.
Nets let us see each face so that we can calculate their area. However, **we can also use a formula to represent the faces as we find their area.** Let’s look again at our calculations for the rectangular prism we dealt with.

<table>
<thead>
<tr>
<th>Bottom face</th>
<th>Top face</th>
<th>long side</th>
<th>long side</th>
<th>short side</th>
<th>short side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = lw )</td>
<td>( A = lw )</td>
<td>( A = lh )</td>
<td>( A = lh )</td>
<td>( A = wh )</td>
<td>( A = wh )</td>
</tr>
<tr>
<td>12 ( \times ) 7</td>
<td>12 ( \times ) 7</td>
<td>12 ( \times ) 3</td>
<td>12 ( \times ) 3</td>
<td>7 ( \times ) 3</td>
<td>7 ( \times ) 3</td>
</tr>
<tr>
<td>84</td>
<td>84</td>
<td>36</td>
<td>36</td>
<td>21</td>
<td>21</td>
</tr>
</tbody>
</table>

\[
SA = 2\times 84 + 2\times 36 + 2\times 21 = 282 \text{ in.}^2
\]

Notice that our calculations repeat in pairs. **This is because every face in a rectangular prism is opposite a face that is congruent.** In other words, the top and bottom faces have the same measurements, the two long side faces have the same measurements, and the two short side faces have the same measurements. Therefore we can create a formula for surface area that gives us a short cut. We simply double each calculation to represent a pair of faces.

**The formula looks like this.**

\[
SA = 2lw + 2lh + 2hw
\]

In this rectangular prism, \( l = 12 \) inches, \( w = 7 \) inches, and \( h = 3 \) inches. We simply put these numbers into the formula and solve for surface area. Let’s try it.

\[
SA = 2(12 \times 7) + 2(12 \times 3) + 2(3 \times 7) \\
SA = 2(84) + 2(36) + 2(21) \\
SA = 168 + 72 + 42 \\
SA = 282 \text{ in.}^2
\]

As we have already seen, the surface area of this prism is 282 square inches. This formula just saves us a little time by allowing us to calculate the area of pairs of faces at a time.

Here is another one.

What is the surface area of the figure below?

All of the faces of this prism are rectangles, so we can use our formula for finding the surface area of rectangular prisms. We simply put the measurements into the formula and solve for \( SA \), surface area.
This rectangular prism has a surface area of 938 square centimeters. If you’re not sure which measurements go with which side of the prism, try drawing a net.

Let’s practice using this formula. Try a few problems on your own.

**Example A**

Length of 8 in, width of 4 inches, height of 6 inches

**Solution:** 208 in²

**Example B**

Length of 5 ft, width of 4 ft, height of 2 ft

**Solution:** 108 ft²

**Example C**

Length of 11 m, width of 8 m, height of 7 m

**Solution:** 442 m²

Here is the original problem once again.

On her first day wrapping boxes, Candice had a customer come up to the counter with a huge box in her arms.

“This is a doll house for my niece,” the woman said. “Can you please wrap it?”
“Certainly,” Candice said looking at the box. The box was $40'' \times 23'' \times 32''$. Candice looked at the different rolls of wrapping paper. She was sure that she was going to need the largest roll of wrapping paper that she could find, and even then she was sure that it was going to take two pieces of paper. The wrapping station has a huge roll of paper on a big roller. This one was brand new.

“How big is this roll?” Candice asked Mrs. Scott.

“It is $24'' \times 900 \text{ feet}$,” Mrs. Scott said.

“That should be big enough,” Candice thought.

To figure out if the wrapping paper on the roll will be enough to wrap the doll house, Candice has to figure out the surface area of the box. She knows the dimensions, so she can use these dimensions to help solve the problem.

The box is $40'' \times 23'' \times 32''$.

Since a box is a rectangular prism, especially given the length and width, we can use this formula for the surface area of the box.

$SA = 2(lw + lh + wh)$

Next, we can substitute the given measurements into the formula.

\[
SA = 2 \cdot [40(23) + 40(32) + 23(32)] \\
SA = 2(920 + 1280 + 736) \\
SA = 2(2936) \\
SA = 5872 \text{ sq.inches}
\]

Let’s change that into square feet by dividing by 12.

$5872 \div 12 = 489.33 \text{ sq.feet}$

We can round up to 490 square feet just to be sure.

Now the measurement of the wrapping paper was in inches and feet.

$24'' \times 900 \text{ ft, so let’s change it to feet}$

$2 \text{ ft} \times 900 \text{ ft} = 1800 \text{ sq.feet}$

The wrapping paper will be enough to cover the box of the doll house.

**Vocabulary**

Here are the vocabulary words in this Concept.

**Prism**

a three-dimensional solid with two congruent parallel bases.

**Area**

the space enclosed inside a two-dimensional figure.

**Surface Area**

the covering of a three dimensional solid.
Net
a two-dimensional representation of a three-dimensional solid.

Rectangular Prism
a prism with rectangles as bases and faces.

Guided Practice

Here is one for you to try on your own.

Crystal is wrapping the box below in wrapping paper for her brother’s birthday. How much wrapping paper will she need?

Answer

First of all, is this a rectangular or triangular prism? All of the faces are rectangles, so it is a rectangular prism. The picture clearly shows us what its length, width, and height are, so let’s use the formula for finding the surface area of rectangular prisms. Simply put the measurements in for the appropriate variables in the formula.

\[
SA = 2lw + 2lh + 2hw
\]
\[
SA = 2(12 \times 9) + 2(12 \times 6) + 2(6 \times 9)
\]
\[
SA = 2(108) + 2(72) + 2(54)
\]
\[
SA = 216 + 144 + 108
\]
\[
SA = 468 \text{ in}^2
\]

Crystal will need 468 square inches of wrapping paper in order to cover the present.

Video Review

Here is a video for review.

<media class="youtube" id="
- This is a video on surface area of rectangular prisms.

## Practice

Directions: Use the formula for surface area to find the surface area of each rectangular prism.

1. A rectangular prism with a length of 10 in, width of 8 in and height of 5 inches.
2. A rectangular prism with a length of 8 in, width of 8 in and height of 7 inches.
3. A rectangular prism with a length of 12 m, width of 4 m and height of 6 meters.
4. A rectangular prism with a length of 10 in, a width of 6 in and a height of 7 inches.
5. A rectangular prism with a length of 12 m, a width of 8 m and a height of 5 meters.
6. A rectangular prism with a length of 9 ft, a width of 7 feet and a height of 6 feet.
7. A rectangular prism with a length of 10 m, a width of 8 m and a height of 2 m.
8. A rectangular prism with a length of 6 ft, a width of 5 feet and a height of 3 feet.
9. A rectangular prism with a length of 3 feet, a width of 6 feet and a height of 2 feet.
10. A rectangular prism with a length of 4 feet, a width of 4 feet and a height of 4 feet.
11. A rectangular prism with a length of 12 feet, a width of 9 feet and a height of 7 feet.
12. A rectangular prism with a length of 14 feet, a width of 11 feet and a height of 10 feet.
13. A rectangular prism with a length of 18 feet, a width of 16 feet and a height of 12 feet.
14. A rectangular prism with a length of 22 meters, a width of 18 meters and a height of 10 meters.
15. A rectangular prism with a length of 21 meters, a width of 18 meters and a height of 17 meters.