Surface Area of Triangular Prisms

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Here you’ll learn to find the surface area of triangular prisms using formulas.

Have you ever seen a triangular prism?

Jesse bought a kaleidoscope for his sister for her birthday. Here is a picture of its shape.

To wrap this figure, Jesse will need to cover it with wrapping paper.

What is the surface area of this triangular prism?

This Concept is about figuring out the surface area of triangular prisms. By the end of the Concept you will know how to figure this out.

**Guidance**

Remember that a **prism** is a three-dimensional object with two congruent parallel bases. The shape of the base names the prism and there are rectangles for the sides of the prism.

When we worked with two-dimensional figures, we measured the area of those figures. The **area** is the space that is contained in a two-dimensional figure. Now we are going to look at the area of three-dimensional figures. Only this isn’t called simply area anymore, it is called surface area.

**What is surface area?**

The **surface area** is the covering of a three-dimensional figure. Imagine you could wrap one of the figures above in wrapping paper, like a present. The amount of wrapping paper needed to cover the figure represents its surface area.

To find the surface area, we must be able to calculate the area of each face and then add these areas together.

One way to do this is to use a net. Remember that a **net** is a two-dimensional representation of a three-dimensional solid. A net is a stretched out picture or an unfolded picture of a solid.

If we look at a net and find the sum of each surface of the net and then add up each measurement, then we will know the measurement of the “cover” of the figure.

What is the surface area of the figure below?
The first thing we need to do is draw a net. Get ready to exercise your imagination! It may help to color the top and bottom faces to keep you on track. Begin by drawing the bottom face. It is a triangle. Each side of the face is connected to a side face. What shape is each side face? They are rectangles, so we draw rectangles along each side of the triangular base. Lastly, we draw the top face, which can be connected to any of the side faces.

Next let’s fill in the measurements for the sides of each face so that we can calculate their area. Be careful! This time two of the faces are triangles. Remember, we calculate the area of triangles with the formula \( A = \frac{1}{2}bh \). We need to know the height of the triangles, look at the diagram to find it. We are going to find the areas of two triangles and three rectangles.

Now that we have the measurements of all the faces, let’s calculate the area of each. Remember to use the correct area formula.

\[
\begin{align*}
\text{Bottom face} & \quad \text{Top face} & \quad \text{side} & \quad \text{side} & \quad \text{side} \\
amp;A = \frac{1}{2}bh & \quad A = \frac{1}{2}bh & \quad A = lw & \quad A = lw & \quad A = lw \\
\frac{1}{2}(8)(6) & \quad + & \quad \frac{1}{2}(8)(6) & \quad + & \quad 17 \times 10 & \quad + & \quad 17 \times 10 & \quad + & \quad 17 \times 8 \\
24 & \quad + & \quad 24 & \quad + & \quad 170 & \quad + & \quad 170 & \quad + & \quad 136 & = & \quad 524 \text{ cm}^2
\end{align*}
\]

We used the formula \( A = \frac{1}{2}bh \) to find the area of the top and bottom faces. We used the formula \( A = lw \) to find the area of the three side faces.

When we add these together, we get a surface area of 524 square centimeters for this triangular prism.

Triangular prisms have a different formula for finding surface area because they have two triangular faces opposite each other. Remember, the formula for the area of triangles is not the same as the area formula for rectangles, so we’ll have to proceed differently here to find a formula for surface area.
First, we know that we need to find the area of the two triangular faces. Each face will have an area of \( \frac{1}{2}bh \). Remember, we can use a formula to calculate the area of a pair of faces. Therefore we can double this formula to find the area of both triangular faces at once. This gives us \( 2(\frac{1}{2}bh) \). The numbers cancel each other out, and we’re left with \( bh \). That part was easy!

Next, we need to calculate the area of each side face. The length of each rectangle is the same as the height of the prism, so we’ll call this \( H \). The width of each rectangle is actually the same as each side of the triangular base. Instead of multiplying the length and width for each rectangle, we can combine this information. We can multiply the perimeter of the triangular base, since it is the sum of each “width” of a rectangular side, by the height of the prism, \( H \).

If we put these pieces together—the area of the bases and the area of the side faces—we get this formula.

\[
SA = bh + (s_1 + s_2 + s_3)H
\]

To use this formula, we fill in the base and height of the prism’s triangular base, the lengths of the base’s sides, and the height of the prism.

Don’t confuse the height of the triangular base with the height of the prism!

Now try a few of these on your own.

**Example A**

A triangular prism has a triangular end with a base of 5 inches and a height of 4 inches. The length of each side is 8 inches and the width of each side is 6 inches. What is the surface area of the prism?

Solution: 164 in\(^2\)

**Example B**

A triangular prism has a triangular end with a base of 8 feet and a height of 6 feet. The length of each side is 10 feet and the width of each side is 6 feet. What is the surface area of the prism?

Solution: 228 ft\(^2\)

**Example C**

A triangular prism has a triangular end with a base of 12 inches and a height of 9 inches. The length of each side is 14 inches and the width of each side is 9 inches. What is the surface area of the prism?

Solution: 486 in\(^2\)
Here is the original problem once again.

Jesse bought a kaleidoscope for his sister for her birthday. Here is a picture of its shape.

![Triangular Prism Diagram]

To wrap this figure, Jesse will need to cover it with wrapping paper.

What is the surface area of this triangular prism?

To figure this out, we have all of the measurements we need. Let’s put them into the formula and solve for surface area, $SA$.

\[
SA = bh + (s_1 + s_2 + s_3)H
\]
\[
SA = 4(3) + (5 + 5 + 4)(15)
\]
\[
SA = 12 + 14(15)
\]
\[
SA = 12 + 210
\]
\[
SA = 222 \text{ cm}^2
\]

This triangular prism has a surface area of 222 square centimeters.

**Vocabulary**

Here are the vocabulary words in this Concept.

**Prism**
- a three-dimensional solid with two congruent parallel bases.

**Area**
- the space enclosed inside a two-dimensional figure.

**Surface Area**
- the covering of a three-dimensional solid.

**Triangular Prism**
- a prism with triangles as bases and rectangles as faces.
Guided Practice

Here is one for you to try on your own.

Find the surface area of this triangular prism.

\[
\text{SA} = \text{Area of three rectangles} + \text{Area of two triangles}
\]
\[
\text{SA} = 2(8 + 9 + 7) + 2 \left( \frac{1}{2} (8)7 \right)
\]
\[
\text{SA} = 2(24) + 2(28)
\]
\[
\text{SA} = 48 + 56
\]
\[
\text{SA} = 104 \text{ sq. in.}
\]

This is our answer.

Video Review

Here is a video for review.

- This is a video on surface area of triangular prisms.

Practice

Directions: Use the following figure to answer each question.
1. What unit are the measurements of this figure?
2. What is the length of the base?
3. What is the width of the base?
4. What is the shape of the sides?
5. What is the measure of the triangular side?
6. What is the formula for finding the surface area of a triangular prism?
7. What is the surface area of this figure?

Directions: Find the surface area of each triangular prism.

8. A triangular prism has a triangular end with a base of 4 inches and a height of 3 inches. The length of each side is 5 inches and the width of each side is 3 inches. What is the surface area of the prism?
9. A triangular prism has a triangular end with a base of 4 feet and a height of 5 feet. The length of each side is 6 feet and the width of each side is 5 feet. What is the surface area of the prism?
10. A triangular prism has a triangular end with a base of 3 inches and a height of 3 inches. The length of each side is 4 inches and the width of each side is 3 inches. What is the surface area of the prism?
11. A triangular prism has a triangular end with a base of 6 meters and a height of 5 meters. The length of each side is 8 meters and the width of each side is 5 meters. What is the surface area of the prism?
12. A triangular prism has a triangular end with a base of 15 feet and a height of 10 feet. The length of each side is 12 feet and the width of each side is 10 feet. What is the surface area of the prism?
13. A triangular prism has a triangular end with a base of 9 inches and a height of 7 inches. The length of each side is 9 inches and the width of each side is 7 inches. What is the surface area of the prism?
14. A triangular prism has a triangular end with a base of 8 inches and a height of 6 inches. The length of each side is 15 inches and the width of each side is 6 inches. What is the surface area of the prism?
15. A triangular prism has a triangular end with a base of 5.5 feet and a height of 4 feet. The length of each side is 9 feet and the width of each side is 5 feet. What is the surface area of the prism?